Modified Time-Domain-Scattering-Parameter Formulation for Incident and Reflected Waves on Lossless, Nonuniform Transmission Lines

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Abstract—**A DC steady state voltage is decomposed into two waves traveling in opposite directions of a lossless, nonuniform transmission line. Such a two-traveling-wave representation for a DC charged line is employed to modify the time-domain-scattering-parameter formulation of a lossless, nonuniform transmission line having nonzero initial condition as well as nonlinear terminations.**

I. Introduction

The TIME-DOMAIN scattering-parameter approach has been widely used to analyze the transients of transmission line terminated with nonlinear loads [1]–[3]. However, most of the studies [1]–[5] have examined wave interactions between transmission lines and nonlinear loads when signal lines are assumed to be initially uncharged. In many practical applications, the omission of dc charges on the signal line cannot represent the real operating condition of circuits. It is therefore pertinent to obtain generalized time-domain-scattering-parameter representation to overcome such a shortcoming. In this letter, a dc steady-state voltage is treated as two waves traveling in opposite directions of the signal line [6]. The two-traveling-wave approach is then employed to derive new time-domain-scattering-parameter formulation, which dictates transient behavior of the nonuniform line having nonzero dc steady-state voltage as well as nonlinear terminations.

II. Modified Time-Domain-Scattering-Parameter Formulation

By taking the inverse Laplace's transforms of frequency-domain-scattering-parameter representation relating incident, reflected waves, and scattering parameters of a two-port network, we obtain the time-domain-scattering-parameter formulation as follows [1]:

$$b_1(t) = S_{11}(t)^* a_1(t) + S_{12}(t)^* a_2(t), \tag{1a}$$

$$b_2(t) = S_{21}(t)^* a_1(t) + S_{22}(t)^* a_2(t), \tag{1b}$$

where t is the time, * denotes a convolution integral in time domain, $s_{ij}(t)$ $(i, j = 1, 2)$ are time-domain scattering parameters of a two-port network, and $a_1(t), b_1(t), a_2(t), b_2(t)$ are the incident and reflected waves for ports 1 and 2, respectively. Such notations provide no explicit information regarding the initial conditions of circuits. We show in Fig. 1 that a nonuniform line is connected to dc voltage sources V_S, V_L and source resistors R_S and R_L on both sides of signal line. Both switches $S1$ and $S2$ are in place long enough so that a dc steady state condition is built upon the signal line. Recent study [6] of transmission lines in dc steady state has revealed that a dc steady state voltage can be represented by two waves traveling in opposite directions of the signal line. The two traveling waves are

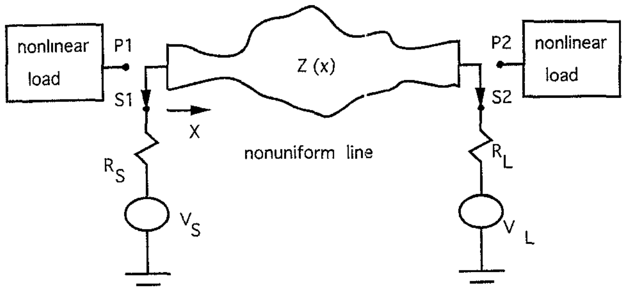

Fig. 1. A nonuniform line connected to dc voltage sources and resistors is under dc steady state condition.

$$V_{+x}(t, x) = \frac{1}{2}[V_{SS} + Z(x)I_{SS}], \tag{2a}$$

$$V_{-x}(t, x) = \frac{1}{2}[V_{SS} - Z(x)I_{SS}], \tag{2b}$$

where x is the space variable, $Z(x)$ is the characteristic impedance at $x, V_{+x}(t, x)$ is the forward traveling wave in $+x$ direction, and $V_{-x}(t, x)$ denotes the backward-traveling wave in $-x$ direction. In (2), V_{SS} and I_{SS} are steady state voltage and current given by

$$V_{SS} = \frac{R_L V_S + R_S V_L}{R_S + R_L}, \tag{3a}$$

$$I_{SS} = \frac{V_S - V_L}{R_S + R_L}, \tag{3b}$$

respectively. Note that V_{SS} and I_{SS} are uniform across the nonuniform transmission line.

When a signal line is under dc steady state condition, the time-domain-scattering-parameter representation appears to be in the form of

Manuscript received February 4, 1994.
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IEEE Log Number 9402177.

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$$b_1(t)\Big|_{\rm dc} = a_1(t)\Big|_{\rm dc}\int_{-\infty}^{\infty} S_{11}(\tau)d\tau + a_2(t)\Big|_{\rm dc}\int_{-\infty}^{\infty} S_{12}(\tau)d\tau, \quad (4a)$$

$$b_2(t)\Big|_{\rm dc} = a_1(t)\Big|_{\rm dc}\int_{-\infty}^{\infty} S_{21}(\tau)d\tau + a_2(t)\Big|_{\rm dc}\int_{-\infty}^{\infty} S_{22}(\tau)d\tau. \quad (4b)$$

Equation (4) presents the quantitative description on how two dc incident waves $a_1(t)|_{\rm dc}, a_2(t)|_{\rm dc}$ at two ends of a signal line relate to the dc-reflected waves $b_1(t)|_{\rm dc}, b_2(t)|_{\rm dc}$ through the time-domain scattering parameters $S_{ij}(t)$. $a_i(t)\big|_{\rm dc}$ and $b_i(t)\big|_{\rm dc}$ are related to the traveling-wave representations of dc steady state line as follows:

$$a_1(t)\Big|_{\rm dc} = V_{x+}(t,0), \quad b_1(t)\Big|_{\rm dc} = V_{x-}(t,0), \quad (5a)$$

$$a_2(t)\Big|_{\rm dc} = V_{x-}(t,L), \quad b_2(t)\Big|_{\rm dc} = V_{x+}(t,L), \quad (5b)$$

where L is the physical length of the nonuniform line. Equation (5) reveals that, under the dc steady state condition, the incident and reflected waves $a_i(t)$ and $b_i(t)$ $(i = 1, 2)$ in time-domain-scattering-parameter representations are equal to the two traveling waves of the dc charged line at the respective terminals, which are shown in (2).

To consider the transients of a nonuniform line terminated with nonlinear loads, we assume that switch $S1$ in Fig. 2 changes its position at $t = 0$. The nonuniform transmission line is represented by its time-domain scattering parameters. The switch $S1$ is connected to source voltage V_S and source resistor R_S long enough so that a dc steady state voltage is built upon the signal line. Right after the closure of switch $S1$ to position $P1$, the incident wave deviates from its dc steady state value in (4a) and (5a). The reflected wave $b_1(t)$ now should be cast in the form of

$$b_1(t) = \int_{-\infty}^{t} S_{11}(t-\tau)\big[a_1(\tau) - a_1(t)\big|_{\rm dc}\big]d\tau + a_1(t)\Big|_{\rm dc} \times \int_{-\infty}^{\infty} S_{11}(\tau)d\tau + a_2(t)\Big|_{\rm dc}\int_{-\infty}^{\infty} S_{12}(\tau)d\tau \quad (6)$$

for $0 < t < t_0$, where $t_0 = L/u$ is the wave propagation delay across the signal line, u is the wave velocity. The difference between $a_1(\tau)$ and $a_1(t)|_{\rm dc}$ represents an incident pulse wave caused by the change of circuit condition on the left-hand side of signal line. For $t > t_0$, the incident wave $a_1(t)$ arrives at the right end of signal line and causes $a_2(t)$ to deviate form its dc steady state value. $b_1(t)$ in (6) should therefore be changed into

$$b_1(t) = \int_{-\infty}^{t} S_{11}(t-\tau)\big[a_1(\tau) - a_1(t)\big|_{\rm dc}\big]d\tau + a_1(t)\Big|_{\rm dc} \times \int_{-\infty}^{\infty} S_{11}(\tau)d\tau + a_2(t)\Big|_{\rm dc}\int_{-\infty}^{\infty} S_{12}(\tau)d\tau + \int_{-\infty}^{t} S_{12}(t-\tau)\big[a_2(\tau) - a_2(t)\big|_{\rm dc}\big]d\tau, \quad (7)$$

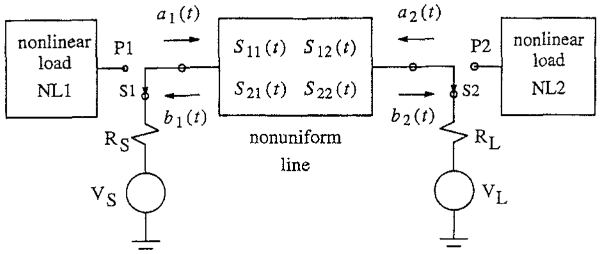

Fig. 2. Time-domain analysis and initial condition considerations for a nonuniform line terminated with nonlinear loads.

for $t > t_0$. Notice that, for $0 < t < t_0$, (7) reduces to (6) since $a_2(\tau) = a_2(t)|_{\rm dc}$. Based on the same reason and the fact of retardation factor in $S_{12}(t)$, we find that the last term of the right-hand side of (7) yields no contribution to $b_1(t)$ until $t > 2t_0$. By the same token, the reflected wave $b_2(t)$ on the right-hand side of signal line should be modified as

$$b_2(t) = \int_{-\infty}^{t} S_{21}(t-\tau)[a_1(\tau) - a_1(t)|_{\rm dc}]d\tau + a_1(t)|_{\rm dc} \times \int_{-\infty}^{\infty} S_{21}(\tau)d\tau + a_2(t)\Big|_{\rm dc}\int_{-\infty}^{\infty} S_{22}(\tau)d\tau + \int_{-\infty}^{t} S_{22}(t-\tau)\big[a_2(\tau) - a_2(t)\big|_{\rm dc}\big]d\tau, \quad (8)$$

for $t > 0$. Both (7) and (8) therefore represent the generalized time-domain-scattering-parameter formulation when initial dc steady-state conditions on a signal line are not zero. As shown in (2) and (5), the values of $a_1(t)|_{\rm dc}$ and $a_2(t)|_{\rm dc}$ in (7) and (8) are equal to those of two traveling waves on a dc steady state line.

III. Conclusion

A dc steady-state voltage is decomposed into two traveling waves propagating in opposite direction of a lossless, nonuniform signal line. Based on the two-traveling-wave representation of a dc charged line, we developed new time-domain-scattering-parameter formulation that dictates both steady-state and transient behaviors of a nonuniform line terminated with nonlinear loads.

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